One-dimension wave propagation

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Objective

Using deep learning algorithms Physics-Informed Neural Networks(PINNs) to solve one-dimension wave equation(1) shown below. By using machine learning methods we hope to achieve a faster and more efficient solver compared to the more traditional numerical solver.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}. \quad (1)$$

Motivation

One of our team members had past experience using the numerical solver to do wave simulations and was unsatisfied with the performance. She wants to use this project as an opportunity to develop an alternative, and hopefully more efficient method for her future wave simulation work.

Deliverable

Our team will deliver a code to find the solution for the wave equation. We will vary the initial conditions to make sure our network is reliable. Plots or animations will be provided to compare our method and the traditional method.

Research Plan

- Our inputs are x and t, where x is the length of the field divided by resolution and t is the time. Our output u is the amplitude of the wave for each x and t.
- We will be using PINNs and the architecture will be similar to Assignment 3. The architecture will consist of several densely connected layers.
- $Loss_{int} \equiv \frac{1}{N}\sum_{i=1}^{N}(\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2})^2$, $Loss_{bc} \equiv \lambda_a(u(0;t;\theta)^2 + u(L;t;\theta)^2)$,
  $Loss_{ic} \equiv \lambda_b(u(x, 0, \theta)^2 - u_{ic})$
- We will use x = linspace(0, L, n_train) as input at each t, train the network to reduce our loss towards zero.
- Our system will be validated by comparing it to the results from the traditional method. We will grid search the hyperparameters to achieve convergence.
- We will test our result by "ground truth" generated from the traditional method.